Levels in Dy^{164} from Deuteron Stripping and Inelastic Proton Scattering Experiments*

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Protons and deuterons with energies 12.5 and 12.0 MeV, respectively, were used to study the levels in Dy^{164} at a number of angles via the reactions $Dy^{164}(p,p')Dy^{164}$ and $Dy^{163}(d,p)Dy^{164}$. The ground-state Q value for the latter reaction was determined as 5434±5 keV. The agreement in energy between the proton groups observed in both reactions and the nuclear models of Bohr and Mottelson and Davydov allows the assignment of ground-state and gamma vibrational bands. These assignments are (in keV) G.S. (0+), 72 (2+), 240 (4+), 503 (6+), 761 (2+), 827 (3+), 919 (4+), 1040 (5+), and, with less certainty, 1157 (6+). Cross sections for excitation of the γ band by the (d,p) reaction are similar to those of the G.S. band, whereas no excitation of a β vibrational band is observed. The 6− two-quasiparticle state, formed from the coupling of the $\frac{5}{2}-$ [523] and $\frac{7}{2}$ [633] neutron states, and the 7− member of the superimposed rotational band are probably the weak states at 1680 and 1853 keV, respectively. The 2+ two-quasiparticle state formed from the coupling of the $\frac{5}{2}-$ [523] and $\frac{1}{2}-$ [521] neutron states and the members of the associated rotational band have probably been observed. They are: 1987 (2+), 2058 (3+), 2158 (4+), 2266 (5+), and 2418 (6+). The collective nature of the ground-state and γ vibrational bands in Dy^{164} is clearly indicated by very small cross sections for the (d,p) reactions and relatively large cross sections for the (p,p') reaction. The energy gap corresponding to breaking a neutron pair is observed as a dramatic increase in the (d,p) cross section. A useful selection rule for (d,p) stripping predicted by Satchler is clearly verified experimentally.

I. INTRODUCTION

THE even-even nucleus Dy^{164} lies toward the middle of the region of strongly deformed rare earth nuclei occurring between the closed neutron shells at 82 and 126. Insofar as it is typical of even-even nuclei in this region, the results which are presented here are believed to be representative of this class of nuclei.

This investigation is concerned first with the general spectroscopic systematics of deformed even-even nuclei and Dy^{164}, in particular, and second, with the striking differences in cross sections for excitation of various states in the (d,p) reaction and for differences between (d,p) and (p,p') excitation of the same state. An attempt will be made to explain both spectroscopic systematics and cross sections in terms of the unified model of the nucleus.

As is often true in reaction spectroscopy, spectroscopic data on Dy^{164} is almost totally missing. In this case, it is at least in part due to the fact that Tb^{164} (a high neutron excess species) has not been synthesized and is presumed to be relatively short-lived. The 73.30 keV 2+ first-excited state of Dy^{164} has been previously observed by Coulomb excitation.[1,2]

II. EXPERIMENTAL APPARATUS AND PROCEDURE

For these experiments a proton beam of 12.5 MeV and a deuteron beam of 12.0 MeV were supplied by the Florida State University Tandem Van de Graaf accelerator.[3] The beam was focused through a series of slits which collimated it to a $\frac{1}{4}$- × 3-mm spot on the target, which then served as a line source for the 60-cm broad-range magnetic spectrograph. Beam currents ranged from 0.4 to 1.2 μA. The acceptance solid angle of the spectrograph was set at 1.78×10^{-4} sr for the (d,p) experiments and 3.56×10^{-4} sr for the (p,p') experiments, where the values given are for the normally deflected ray. The analyzed protons were recorded on an 8-mm × 100-cm strip along a series of 50-μ nuclear track plates forced to assume the shape of the hyperbolic focal curve of the spectrograph. After development, the plates were scanned with a microscope in $\frac{1}{2}$- × 8-mm strips along the exposed region.

The spectrograph was calibrated by placing groups of Po^{210} alpha particles along the focal surface by variation of the magnetic field strength. The validity of the calibration curve for single field strengths was then checked by elastically scattering O^{16} (4+) atoms from a thin Ho^{165} target and simultaneously recording the scattered O^{16} atoms of the same energy, but in various final charge states on the focal surface.

Targets were prepared by first evaporating a thin layer of spectroscopically pure carbon on a glass plate which had been previously prepared with a thin layer of teepol. The carbon layers were 15–30 $\mu g/cm^2$ thick. Enriched isotopes of the metal oxides were then evaporated onto the carbon from a carbon crucible which was heated by electron bombardment. The carbon-backed targets were then floated off the glass in ion-exchanged water and picked up on aluminum target frames having an area of 0.8 cm^2. Despite the great care taken in target preparation, the presence of impurities was a serious handicap in the inelastic scattering experiments. Loss of resolution due to carbon building up on the beam spot of the target was reduced by pumping the target chamber with a mercury diffusion pump through a liquid

* Supported by the U. S. Atomic Energy Commission under Contract AT-(40-1)-2434.
[1] R. L. Chupp, J. W. M. DuMond, F. J. Gordon, R. C. Jopson, and H. Mark, Phys. Rev. 112, 518 (1958).
[2] T. Huus, J. H. Bjerregaard, and B. Elbek, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 30, No. 17, 68 (1956).
[3] Supported in part by the U. S. Air Force Office of Scientific Research under Contract AF AFOSR-62-423 and by the Nuclear Program of Florida State University.

nitrogen trap. In addition, the beam spot was changed at regular intervals throughout the course of a run.

III. DEUTERON STRIPPING

The protons resulting from bombardment of the 74% enriched Dy^{163} target with 12.0-MeV deuterons were analyzed at 25, 35, 45, and 65°, with respect to the incident beam. The spectra for 45 and 65° are shown in Fig. 1 and Fig. 2, respectively. In order to facilitate the track counting, the nuclear emulsion plates were covered with 5 mil aluminum foil to stop the deuterons. The addition of this foil did not measurably affect the resolution of the spectrograph for protons. Dysprosium levels are indicated with numbers and listed in Table I. Eleven of the groups either arise or are contributed to by the reaction $Dy^{164}(d,p)Dy^{165}$, due to the 17.6% Dy^{164} in the target, and three of the groups arise from the reaction $Dy^{162}(d,p)Dy^{163}$ due to the 7.0% Dy^{162}. The identification of the Dy^{165} groups was made from runs on a 90% enriched Dy^{164} target, and the identification of the Dy^{163} groups was made from runs made on an 87% enriched Dy^{162} target.

The determination of the ground-state Q value for the reaction $Dy^{163}(d,p)Dy^{164}$ was made with respect to the Q value for the reaction $C^{13}(d,p)C^{14}$ which is known[4] to be 5951.3±0.8 keV. The Dy^{164} ground-state group was placed to within approximately 2 cm of the C^{14} ground-state group on the nuclear track plates. In this way, systematic errors were essentially eliminated in the determination of the ground-state G value. This determination was made at 25° where the C^{14} ground-state group is intense, and thus the main contribution to the random error is due to the leading edge of the low intensity Dy^{164} ground-state group. The Q value so determined is 5434±5 keV.

Groups 5 and 6 lie so close in energy that they are not completely resolved by the spectrograph. The doublet nature of the resulting group can be inferred both from the shape of the group and the change in shape with observation angle. Impurity groups were identified by their kinematic shift with observation angle.

IV. INELASTIC PROTON SCATTERING

Protons resulting from the bombardment of the 90% enriched Dy^{164} target with 12.5-MeV protons were analyzed at 90 and 133° with respect to the incident beam. The spectrum for 133° is shown in Fig. 3. The resolution of these spectra is poor due to the target which was made purposely thick. Proton groups from impurities are indicated above these groups on the graph. Nine groups are observed which arise from elastic and inelastic scattering on Dy^{164} and Dy^{163}. These groups are numbered and listed in Table II. The 161-keV

[4] F. Everling, L. A. Koenig, J. H. E. Mattauch, and A. H. Wapstra, *1960 Nuclear Data Tables* (U. S. Atomic Energy Commission, Washington, D. C., 1961).

TABLE I. Proton groups from deuteron stripping on the Dy^{163} target.

Group number	Residual nucleus	Q (keV)	Excitation (keV)	Estimated random error (keV)
0	Dy^{164}	5434	0	
1	Dy^{164}	5362	72	2
2	Dy^{164}	5194	240	2
3	Dy^{164}	4931	503	3
4	Dy^{164}	4673	761	2
5	Dy^{164}	4607	827	5
6	Dy^{164}	4595	839	4
7	Dy^{164}	4515	919	3
8	Dy^{164}	4458	976	4
9	Dy^{164}	4394	1040	4
10	Dy^{164}	4277	1157	5
11	Dy^{164}	3754	1680	11
12	Dy^{164}	3708	1726	7
13	Dy^{163}	3688		
14	Dy^{163}	3623		
15	Dy^{164}	3581	1853	8
16	Dy^{163}	3518		
17	Dy^{164}	3496	1938	3
18	Dy^{164}	3447	1987	2
19	$Dy^{164,165}$	3376	2058	4
20	Dy^{164}	3353	2081	4
21	Dy^{164}	3308	2126	2
22	Dy^{164}	3276	2158	2
23	Dy^{165}	3232	2202	
24	$Dy^{164,165}$	3186		
25	Dy^{164}	3168	2266	4
26	Dy^{164}	3128	2306	3
27	Dy^{164}	3094	2340	6
28	Dy^{164}	3076	2358	6
29	Dy^{164}	3016	2418	5
30	Dy^{164}	2956	2478	4
31	Dy^{165}	2884		
32	Dy^{164}	2847	2587	4
33	Dy^{164}	2774	2660	2
34	Dy^{164}	2740	2694	4
35	Dy^{164}	2712	2722	3
36	Dy^{164}	2673	2761	4
37	Dy^{164}	2631	2803	3
38	Dy^{164}	2602	2832	3
39	Dy^{164}	2582	2852	6
40	Dy^{164}	2544	2890	5
41	Dy^{164}	2516	2918	6
42	Dy^{164}	2416	3018	5
43	Dy^{164}	2384	3050	4
44	Dy^{165}	2324		
45	Dy^{164}	2287	3147	4
46	Dy^{164}	2243	3191	3
47	Dy^{164}	2223	3211	4
48	Dy^{164}	2195	3239	3
49	Dy^{165}	2175		
50	Dy^{165}	2150		
51	Dy^{165}	2104		
52	Dy^{164}	2080	3354	8
53	Dy^{164}	2043	3391	5
54	Dy^{164}	2005	3429	5
55	Dy^{165}	1981		
56	Dy^{164}	1958	3476	9
57	Dy^{165}	1925		
58	Dy^{165}	1890		

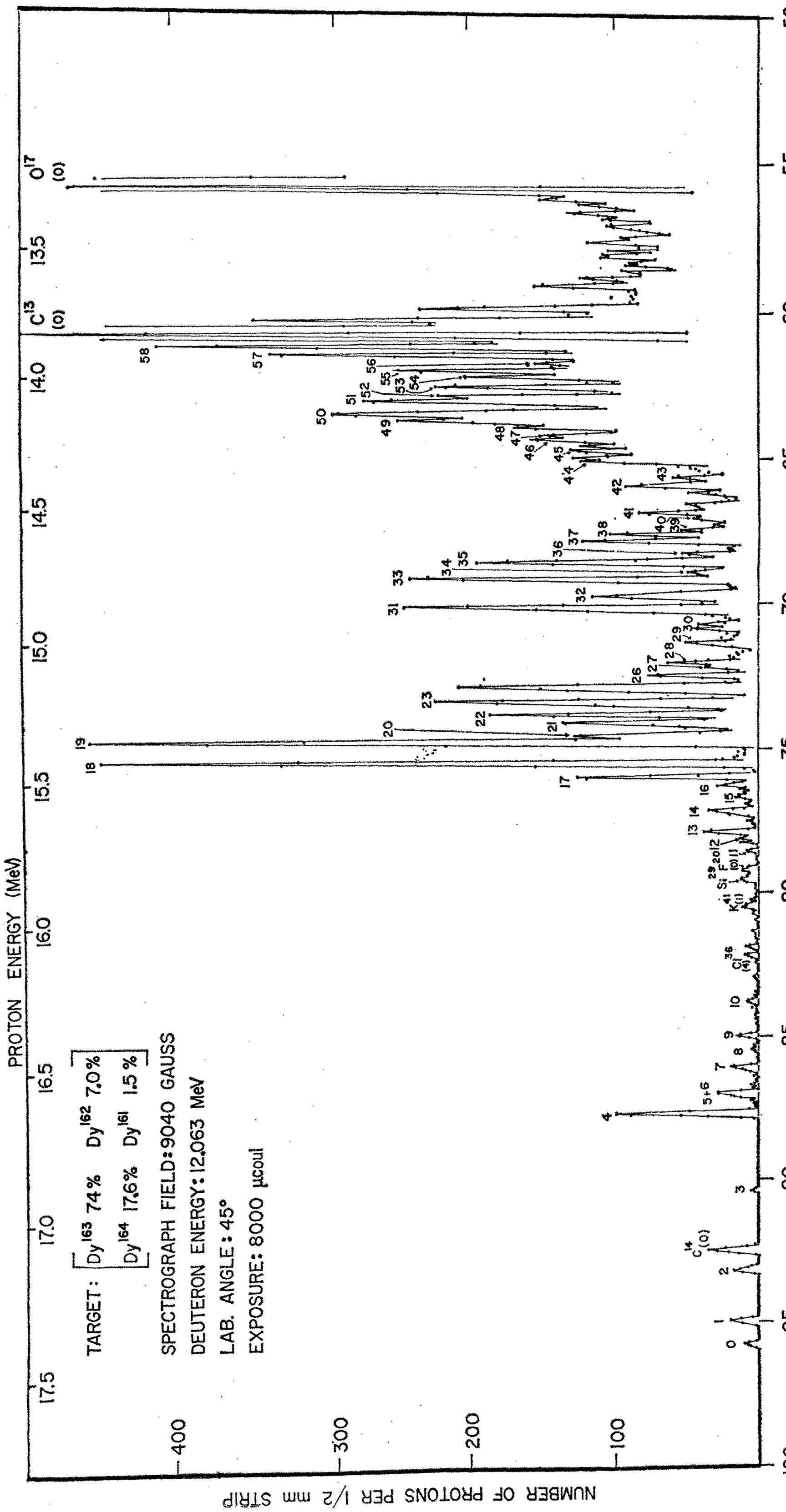

FIG. 1. Deuteron stripping on Dy^{163} at 45°.

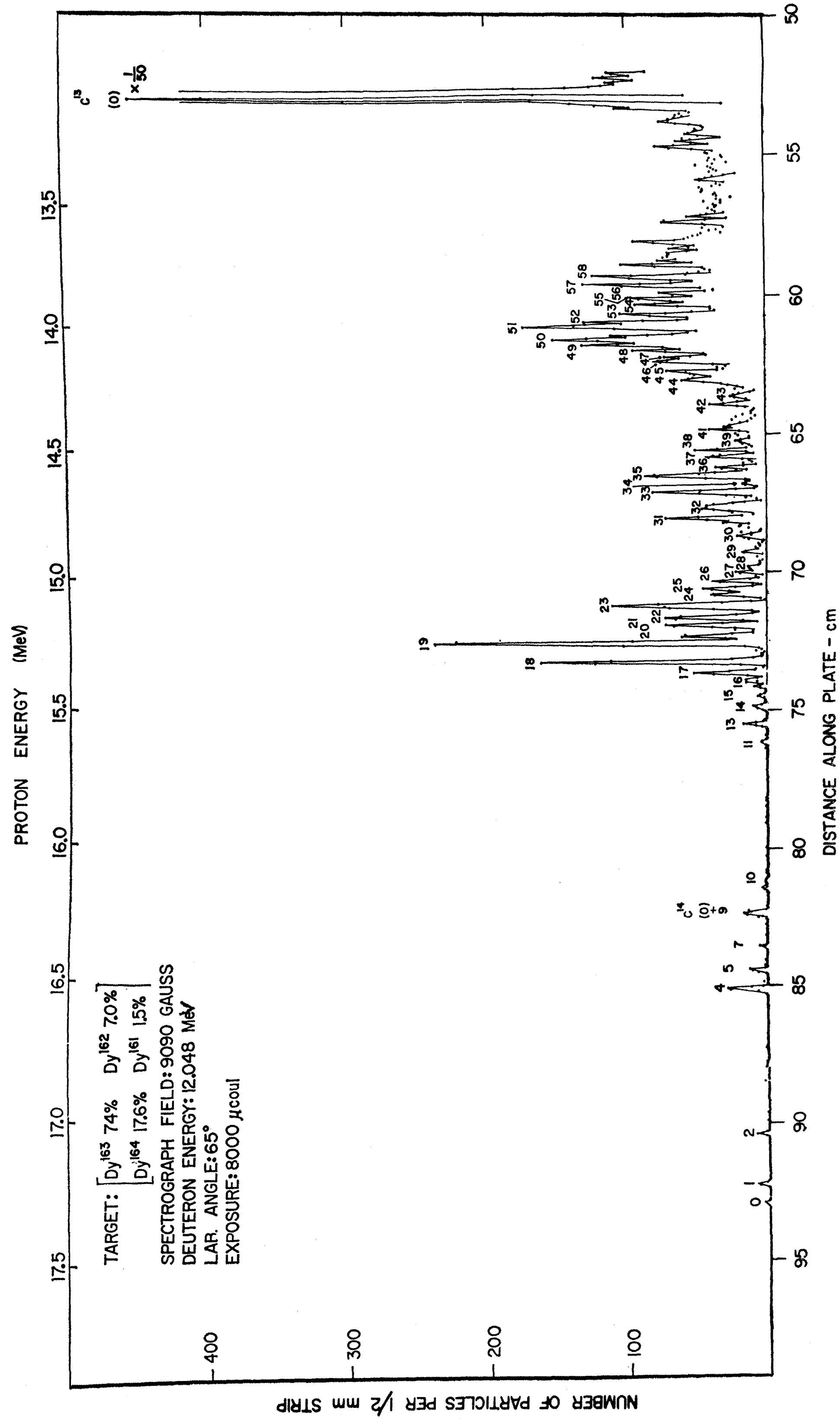

FIG. 2. Deuteron stripping on Dy^{163} at 65°.

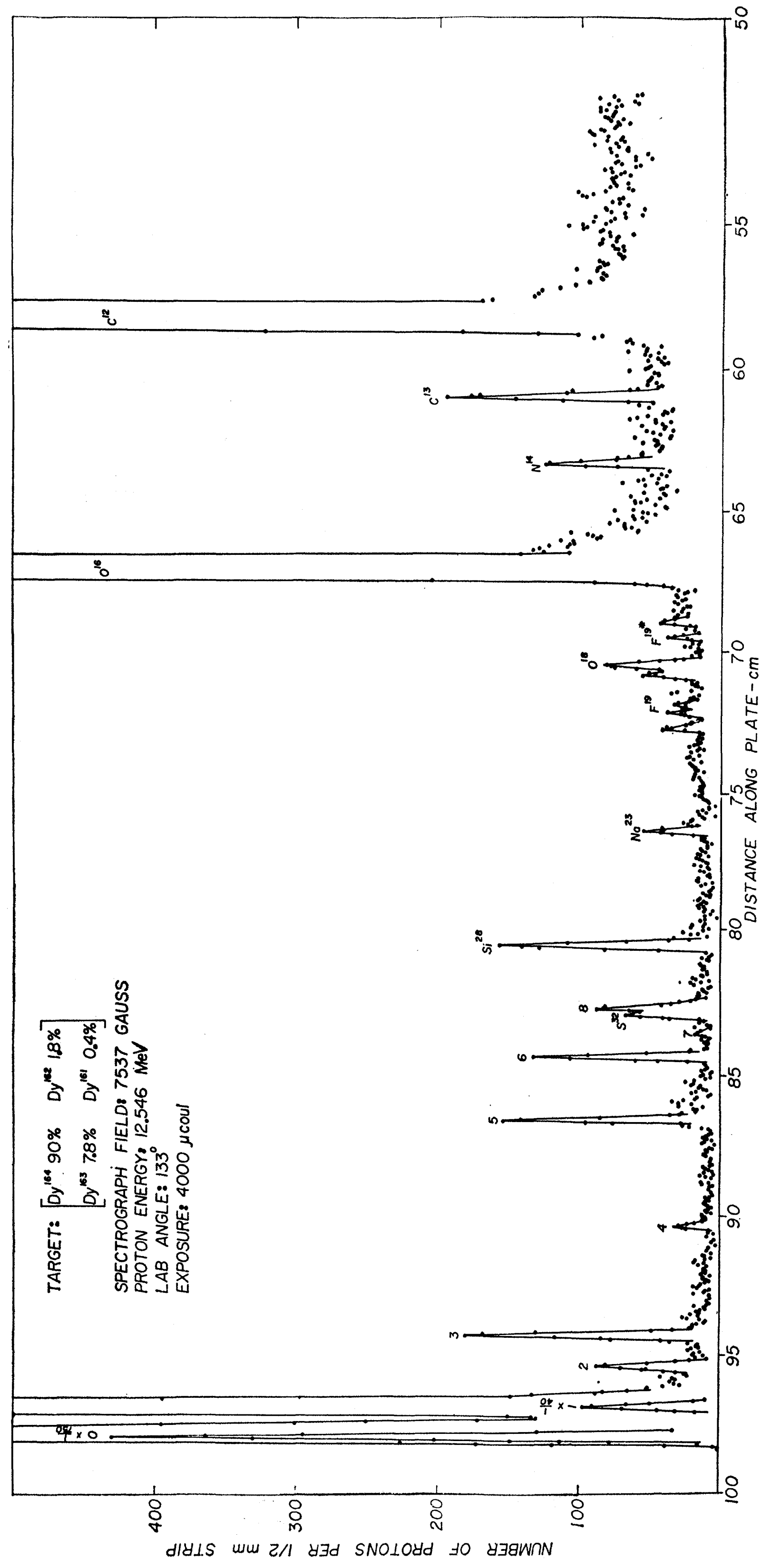

FIG. 3. Elastic and inelastic scattering of protons on Dy^{164} at 133°.

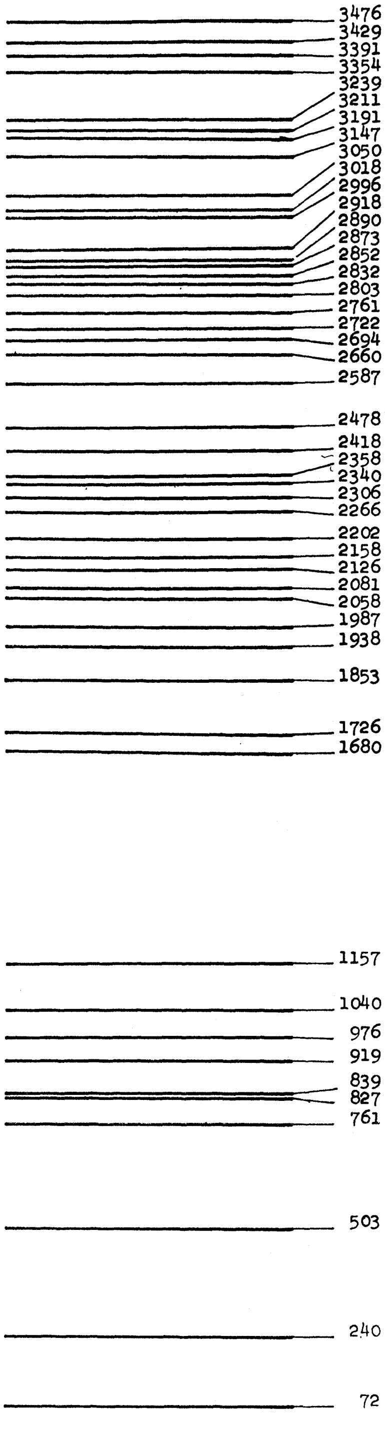

FIG. 4. Level diagram for Dy^{164}.

TABLE II. Inelastic proton groups from the Dy^{164} target.

Group number	Excitation (keV)	Error (keV)	Isotope
0	0		Dy^{164}, Dy^{163}
1	72	2	Dy^{164}
2	161	10	Dy^{163}
3	240	2	Dy^{164}
4	503	3	Dy^{164}
5	761	2	Dy^{164}
6	919	3	Dy^{164}
7	976	4	Dy^{164}
8	1040	4	Dy^{164}

group is a known $\frac{9}{2}$-level[5] in Dy^{163} which has been previously measured as 170 keV. The discrepancy is due to the fact that in this work this group sits on the tail of the intense elastic scattering group of Dy^{164}. The group at 72 keV is essentially the Coulomb-excited 73.30-keV 2+ first excited state of Dy^{164}, and contains a small contribution from the 74 keV, $\frac{7}{2}$-Coulomb-excited first excited state of Dy^{163}. The 72, 240, 503-keV groups fall into a rotational band, and the latter two states are assigned spins and parities of 4+ and 6+ on this basis. The 761-, 919-, and 1040-keV levels are shown below to be part of a rotational band, and assignments are made for these levels also. The fact that these six levels, along with the level at 976 keV, belong to Dy^{164}, follows unambiguously from their appearance in the (d,p) spectra.

V. SPECTROSCOPIC DISCUSSION OF THE DATA

The level diagram constructed from the experimental data is shown in Fig. 4. The value of the rotational constant, $\hbar^2/2\mathfrak{J}$, for the ground-state rotational band is found to be 11.95 keV by fitting the experimentally determined levels to the equation

$$E=\hbar^2/2\mathfrak{J}I(I+1)-BI^2(I+1)^2.$$

The constant B is found to be zero to within the experimental error, and on the basis of an estimate, this error is believed to be less than 0.003.

A second rotational band begins with the level at 761 keV and contains levels at 827, 919, 1040, and possibly the level at 1157 keV. According to the unified model, the spacings of the rotational levels built on a band head with spin 2 and energy E should go as

$$E=E_0+\hbar^2/2\mathfrak{J}[I(I+1)-I_0(I_0+1)].$$

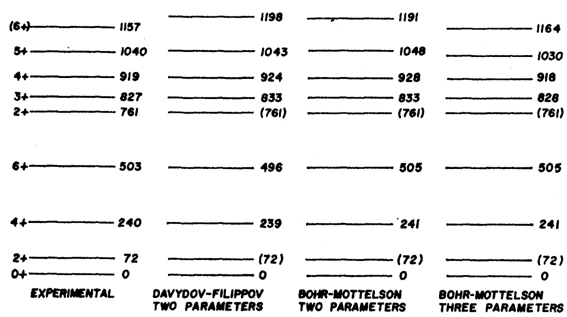

FIG. 5. Comparisons of the experimental data with Bohr-Mottelson and asymmetric rotor models.

[5] N. P. Heydenburg and G. F. Pieper, Phys. Rev. **107**, 1297 (1957).

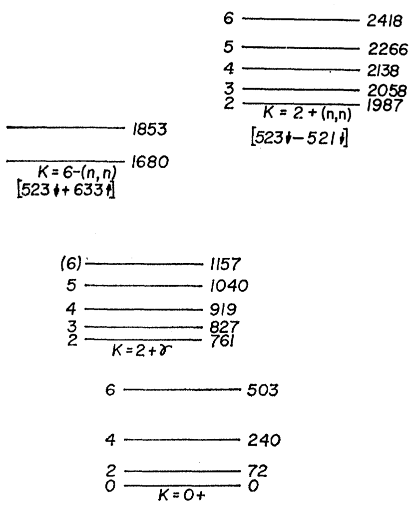

FIG. 6. Rotational bands in Dy^{164}.

On this basis, the level at 761 keV (where I_0 is the energy of the 2+ band head) has spin and parity 2+; that at 827, 3+; the 919 level, 4+; and the 1040 level, 5+. There is some possibility that the level at 1156 is the 6+ member of the rotational band. However, the energy fit makes this assignment less certain than the other assignments. Because of the spin sequence, this rotational band is identified with the $K=2$ gamma band of the Bohr-Mottelson[6] collective model. Figure 5 shows the fit of the experimental data to the Bohr-Mottelson model. Three parameters are used to fit the data. These are: (1) the moment of inertia of the ground-state rotational band, (2) the position of the gamma vibrational state at 761 keV, and (3) the moment of inertia of the rotational band superimposed on the gamma vibration.

Figure 5 also shows the fit to the asymmetric rotor model of Davydov[7] in which the gamma band occurs as a natural part of the ground-state rotational band of a nucleus which deviates from axial symmetry. The extent of the deviation from axial symmetry is characterized by the "nonaxiality" parameter, γ. The value of γ was determined as 12.3° from the ratio of the energy of the second 2+ level to that of the first 2+ level. A set of energy ratio tables for this model calculated on a digital computer by White *et al.* was used. The model of Davydov has been extended by Davydov and Chaban[8] to include exactly the rotation-vibration interaction with a parameter μ. The best fit to Dy^{164} was obtained with no rotation-vibration correction, corresponding to $\mu=0$.

Both models fit the data well up to the level at 1040 keV. It is interesting that the Davydov model fits the data with two parameters (corresponding to the energies of the two 2+ levels) almost as well as the Bohr-Mottelson model fits it with three.

[6] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16, 22 (1953).
[7] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).
[8] A. S. Davydov and A. A. Chaban, Nucl. Phys. **20**, 499 (1960).

In addition to the collective rotational bands described above, we have been able to identify rotational bands built on two-quasiparticle neutron states. Additional information for this identification was obtained by running the reaction $Dy^{164}(d,p)Dy^{165}$. The low-lying one-quasiparticle neutron states in Dy^{165} were identified and the relative cross sections were determined for these states and their superimposed rotational bands. From this information we were able to estimate roughly the relative positions of the two-quasiparticle neutron states in Dy^{164} as well as their relative intensities.

The 1− and 6− two-quasiparticle states formed from anti-parallel and parallel coupling of the $\frac{5}{2}-$ [523] orbital (ground state of Dy^{163}) with the $\frac{7}{2}+$ [633] orbital (ground state of Dy^{165}) are expected to lie lowest and be of very low intensity. The j,l values contributing to the excitation of these states and their superimposed rotational bands are expected to be $j=\frac{9}{2}$, $l=4$ and $j=\frac{13}{2}$, $l=6$. We believe that group 11 is the 6− state and that group 15 is the 7− member of the superimposed rotational band. The energies are 1680 and 1853 keV, respectively. (See Fig. 6.) These assignments are to be considered tentative.

The 2+ and 3+ two-quasiparticle states formed from the coupling of the $\frac{5}{2}-$ [523] orbital (ground state of Dy^{163}) with the $\frac{1}{2}-$ [521] orbital (108-keV excited state of Dy^{165}) are expected to lie next highest in energy. Group 18 is believed to be the 2+ two-quasiparticle state, and groups 19, 22, 25, and 29 are believed to be the 3+, 4+, 5+, and 6+ members of the superimposed rotational band. The energies are 1987, 2058, 2158, 2266, and 2418, respectively. (See Fig. 6.)

VI. RELATIVE CROSS SECTIONS

Calculation of the spectroscopic factors for stripping into a rotational band of a deformed nucleus have been made by Satchler.[9] The l dependence can be obtained from a distorted wave Born approximation calculation. We have made comparisons of the relative intensities of members of rotational bands using Nilsson wave functions and roughly estimated l dependence and find good agreement. Such a comparison is possible since the variations of the intrinsic stripping probabilities are often very large and quite random, whereas the dependence on l is generally slow and smooth. These comparisons have been useful for checking the intensities of members of rotational bands built on two-quasiparticle neutron states.

The variation in the stripping cross sections with the change in the nature of the states excited is striking. It is clear that the (d,p) cross sections for exciting collective states are quite small compared to the cross sections for exciting the majority of the single-particle states. This is because the wave functions for the complicated collective states do not resemble those of the target

[9] G. R. Satchler, Ann. Phys. (Paris) **3**, 275 (1958).

nucleus plus a neutron nearly so much as do the two-quasiparticle neutron states. The energy gap is vividly demonstrated by the very rapid increase in cross section after the breaking of the first neutron pair. It is to be emphasized that the energy gap so obtained is not necessarily the true energy gap, since a proton pair may break at lower energy, and proton states are not appreciably excited by the (d,p) reaction. There is reason to believe that the lowest two-quasiparticle state in Dy^{164} should be a proton state.

It is interesting to note that whereas the ground-state rotational band and gamma vibrational band are not easily excited by the (d,p) reaction, these states are excited with the largest cross sections in the (p,p') reaction. In this case the wave function for the target nucleus resembles closely the wave functions for the low-lying collective states, and this enhances the excitation.

It is of special interest to note that the cross sections for the (d,p) reaction to the γ vibrational band are very similar to those of the ground-state band, whereas no excitation of a β vibrational band is observed.

We have been able to test an interesting selection rule pointed out by Satchler.[9] When applied to the ground-state rotational band of a deformed even-even nucleus, this section rule requires that the angular momentum of the transferred neutron must equal or exceed the spin of the target nucleus, even if angular momentum conservation permits lower values. The spin parity of Dy^{163} is $\frac{5}{2}-$ so that the rule requires $j\geq\frac{5}{2}$, $l\geq 3$ for stripping into the ground-state rotational band of Dy^{164}. However, angular momentum conservation permits the $2+$ rotational state to be populated by $j=\frac{1}{2}$, $\frac{3}{2}$, and $l=1$. Any such contribution should be strongly enhanced by the stripping distribution. That such a contribution is very small, if present, can be shown in two ways. First, the ratio of the intensity of the $2+$ to the $0+$ is constant to within statistics at the several angles measured, which indicates that the $2+$ is populated essentially by $l=3$. Second, the $j=\frac{5}{2}$ contribution to the $2+$ can be determined directly from the $0+$ intensity using the calculations of Satchler; the two are related by a Clebsch-Gordan coefficient. The result is that 76% of the population of the $2+$ is due to $j=\frac{5}{2}$. The other 24% is a reasonable contribution for $j=\frac{7}{2}$ along with a small possible contribution from $j=\frac{9}{2}$. Deviations from the selection rule, if any, are small.

[*Note added in proof.* Recent neutron capture gamma-ray data have been made available to us by Dr. Otto Schult, Danish Atomic Energy Commission, Roskilde, Denmark, on the reaction $Dy^{163}(n,\gamma)Dy^{164}$. He finds the $5+$ rotational state built on the gamma vibration at an energy of 1024.63 keV. Careful re-examination of our data shows the existence of this state also. When this correction is made, the state tentatively assigned as $6+$ at 1157 keV fits the rotational sequence much better, and its assignment is now considerably more certain. Thus, another spin parity assignment of the 1040-keV state is necessary.]

ACKNOWLEDGMENTS

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